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Yield Model Development

EVALUATION OF THE WILLIAMS-TYPE MODEL FOR BARLEY YIELDS IN NORTH DAKOTA AND MINNESOTA

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Evaluation of the Williams-Type Model
for Barley Yields in North Dakota and Minnesota

by

T. L. Barnett

This research was conducted as part of the AgRISTARS Yield Model Development Project. It is part of Task 4 (Subtask 1) in Major Project Element 1 as identified in the Yield Model Development Project Implementation Plan dated March, 1981 (YM-J1-C0618, JSC -- 16357).

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ABSTRACT

The Williams-type yield model is based on multiple regression analysis of historical time series data at CRD level pooled to regional level (groups of similar CRD's). Basic variables considered in the analysis include USDA yield, monthly mean temperature, monthly precipitation, soil texture and topographic information, and variables derived from these. Technological trend is represented by piecewise linear and/or quadratic functions of year. Indicators of yield reliability obtained from a ten-year bootstrap test (1970-1979) demonstrate that biases are small and performance based on root mean square error appears to be acceptable for the intended AgRISTARS large area applications. The model is objective, adequate, timely, simple, and not costly. It considers scientific knowledge on a broad scale but not in detail, and does not provide a good current measure of modeled yield reliability.

Key words: Model evaluation, yield modeling, linear regression.

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Evaluation of Williams-Type Model
for Barley Yields in
North Dakota and Minnesota

Summary

The Williams-type yield model is based on multiple regression analysis of historical time series data at CRD level pooled to regional level (groups of similar CRD's). Basic variables considered in the analysis include USDA yield, monthly mean temperature, monthly precipitation, soil texture and topographic information, and variables derived from these. Technological trend is represented by piecewise linear and/or quadratic functions of year. Model performance is evaluated on the basis of eight criteria, reliability, objectivity, consistency with scientific knowledge, adequacy, timeliness, cost, simplicity, and accurate current measures of modeled yield reliability. Ten year bootstrap tests (1970-1979) were run for each crop reporting district in the major barley producing regions of North Dakota and Minnesota. Indicators of yield reliability obtained from a ten-year bootstrap test (1970-1979) demonstrate that biases are small and performance based on root mean square error is generally acceptable for the intended AgRISTARS large area applications. The model is objective, adequate, timely, simple, and not costly. It considers scientific knowledge on a broad scale but not in detail, and does not provide a good current measure of modeled yield reliability.

Description of Model

A model for analyzing the effects of weather and soil variable on Canadian barley yields was described by Williams et. al. (G.D.V. Williams, M.I. Joynt, P.A. McCormick, Regression Analysis of Canadian Prairie Crop District Cereal Yields, 1961-1972, in Relation to Weather, Soil, and

Trend, Can. J. Soil Sci. 55, 43-53, February 1975). The models for Canadian wheat, barley and rye pooled crop district weather and agronomic data to larger soil-color regions and incorporated soil texture and topographic information along with trend and weather.

A predictive yield model for barley in North Dakota (ND) and Minnesota (MN), based on the concepts outlined by Williams et. al., was developed and tested by the AgRISTARS Yield Model Development Group. The model incorporated CRD-level weather (monthly mean temperature and total precipitation), soil texture, and topography in a manner as similar as possible to that used by Williams. The CRD-level data were pooled to the following two more-or-less environmentally homogeneous regions:

(a) Red River Valley (MNRR) - consisting of ND CRD's 30 and 60 and MN CRD's 10 and 40;

(b) The remainder of North Dakota (NDREM) - consisting of ND CRD's 10,20,40,50,70,80,90.

Separate models were developed for the two regions to provide predictions of CRD yields using individual CRD weather/soil data with coefficients from the pooled model. Models were also developed for the two states, ND and MN, based on state-aggregated weather/soil data.

Models were developed on the basis of data from 1932 through 1979. The terms were selected from stepwise regressions from which the most significant ten (or fewer) terms were retained for each region. A limit of 10 terms had been used by Williams et. al. and seemed to be a reasonable upper limit in applying this method. The basic weather/soil/trend inputs are:

monthly mean temperature;

total monthly precipitation;

percent of soils in the CRD in textural classes--coarse, medium and fine;

percent of CRD area in the topographic class level to gently undulating;

year as surrogate for technological etc. trend;

These basic inputs are used to calculate the possible model variables

Trend 1 (=1 for 1931,..., 31 for 1961; 32 for 1962 and beyond);

Trend 2 (=0.1 for 1931-1961, 1 for 1962,..., 17 for 1979);

Trend 2 squared;

$Tx = .75(\% \text{ fine soil}) + .65(\% \text{ medium soil}) + .35(\% \text{ coarse soil})$;

Tx squared;

Top = % of area level to gently undulating;

Top squared;

C = precipitation September-April;

C squared:

E5, E6, E7 = potential evapotranspiration calculated by the Thornthwaite method (Thornthwaite, C.W., "An Approach Toward a Rational Classification of Climate," Geog. Rev. 38: 55-94, 1948) for May, June, July;

E5, E6, E7 squared;

$D6, D7^{**} = \text{moisture deficits} = E - \text{precipitation for June, July}$;

D5, D6, D7 squared;

$Do = \text{seasonal deficit} = D5 + D6 + D7 - C$;

Do squared;

$Tx \times Do$

*Trend was chosen to correspond to the CEAS barley yield model (Motha, R.P., Barley Models for North Dakota and Minnesota", NOAA-CEAS, Columbia, MO, May 1980) to permit more direct comparison. Model fits using TREND2 = 1.0 made no significant differences in yield model predictions.

**D5 was not used since D5, D6, D7, C and Do are not all mutually independent.

Of these possible terms, the stepwise regression selected 10 terms or fewer for each region.

Bootstrap tests were conducted to provide ten years of independent tests of each model's predictive performance in a manner simulating very closely the way the models are applied in practice. Appendix 2 shows the terms included in each model and the range of coefficients over ten different but overlapping model base periods associated with the ten test years. There are some general patterns, but a wide diversity in detail, reflecting both real region-to-region variations and vagaries of the regression process on noisy data.

Only end-of-season models were tested, although "truncated" models providing yield estimates at the end of each month throughout the growing season were possible. It was felt that meaningful evaluation was difficult enough when the full-season weather was available.

EVALUATION METHODOLOGY

Eight Model Characteristics to be Discussed

The document, Crop Yield Model Test and Evaluation Criteria, (Wilson, et. al., 1980), states:

"The model characteristics to be emphasized in the evaluation process are: yield indication reliability, objectivity, consistency with scientific knowledge, adequacy, timeliness, minimum costs, simplicity, and accurate current measures of modeled yield reliability."

Each of these characteristics will be discussed with respect to the Williams-type model.

Bootstrap Technique Used to Generate Indicators of Yield Reliability

Indicators of yield reliability (reviewed below) require that the parameters of the regression model be computed for a set of data and that a yield prediction be made based on that data for a given "test" year. The values required to generate indicators of yield reliability include the predicted yield, \hat{Y} , the actual (reported) yield, Y , and the difference bet-

ween them, $d = \hat{Y} - Y$, for each test year. It is desirable that the data used to generate the parameters for the model not include data from the test year.

In order to accomplish this, the "bootstrap" technique is used. For each test year, the years from an earlier base period are used to fit the model and obtain a prediction equation. The values of the independent variables for the test year are inserted into the equation and a predicted yield is generated. Then, the base period is shifted one year forward and the process is repeated. Continuing in this way, ten (1970-79) predictions of yield are obtained, each independent of the data used to fit the model.

The \hat{Y} and d values for the ten year test period are obtained from models derived at the crop reporting district (CRD) level and state level, the latter based on a weighted average of CRD weather to state level. A second set of \hat{Y} values are obtained at the state level using a weighted average of predicted yields from the CRD models, and at regional level using weighted averages of predicted yields from CRD and state models. In each case the weighting factors are harvested acreage for the prediction year.

For the Red River Valley region (MN CRD's 10 and 40, ND CRD's 30 and 60) data from 1932-1969 are used to fit predictive models for 1970 data, from 1932-1970 are used to fit predictive models for 1971, etc. through 1979. For the remainder of North Dakota (ND CRD's 10,20,40,50,70,80 & 90) data from 1948-1969 are used to fit predictive models for 1971, etc. through 1979. The number of observations used for the two pooled models were roughly equivalent; fewer years were used with the remainder of North Dakota but more CRD's were involved. This testing procedure closely simulates the way the models would be applied in practice. Results are listed in Appendix 1.

The average and percent production as well as the average yield over the ten year test period are listed in Table 1 for each geographical region, and percent production is displayed in Figure 1.

Review of Indicators of Yield Reliability

The \hat{Y} , Y and d values for the ten-year test period at each geographic area may be summarized into various indicators of yield reliability.

Indicators Based on d Demonstrate Accuracy, Precision and Bias

From the d value, the mean square error (root and relative root mean square error), the variance (standard deviation and relative standard deviation), and the bias (its square and the relative bias) are obtained.

The root mean square error (RMSE) and the standard deviation (SD) indicate the accuracy and precision of the model and are expressed in the original units of measure (quintals/hectare). It is about 68 percent probable that the absolute value of d for a future year will be less than one RMSE and 95 percent probable that it will be less than twice the RMSE. So, accurate prediction capability is indicated by a small RMSE.

A non-zero bias means the model is, on the average, overestimating the yield (positive bias) or underestimating the yield (negative bias). The SD is smaller than the RMSE when there is non-zero bias and indicates what the RMSE would be if there were no bias. If the bias is near zero, the SD and the RMSE will be close in value. An unbiased model, i.e. bias close to zero, is preferred.

Indicators Based on rd Demonstrate Worst and Best Performance

The relative difference, rd ($100d/Y$), is an especially useful indicator in years where a low actual yield is not predicted accurately. This is because years with small observed actual yields and large differences often have the largest rd values.

Several indicators are derived using relative differences. In order to calculate the proportion of years beyond a critical error limit, we count the number of years in which the absolute value of the relative difference exceeds the critical limit of 10 percent. Values between 5 and 25 percent were investigated and a critical limit of 10 percent was found most useful in describing model performance. The worst and next to worst performance during the test period are defined as the largest and next to largest absolute value of the relative difference. The range of yield indication accuracy is defined by the largest and smallest absolute values of the relative difference.

Indicator Based on \hat{Y} and Y Demonstrate
Correspondence Between Actual and Predicted Yields

Another set of indicators demonstrates the correspondence between actual and predicted yields. It would be desirable for increases in actual yield to be accompanied by increases in predicted yields. It would also be desirable for large (small) actual yields to correspond to large (small) predicted yields.

Two indicators relate the change in direction of actual yields to the corresponding change in predicted yields. One looks at change from the previous year (nine observations) and the other at change from the average of the previous three years (seven observations). A base period of three years is used since a longer base period would further decrease the number of observations, while a shorter period would not be very different from the comparison to a single previous year.

Finally, the Pearson correlation coefficient, r , between the set of actual and predicted values for the test years is computed. This represents a measure of how well deviations in the set of predicted yields correlate to deviations in the set of actual yields. It is desirable that

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$r(-1 \leq r \leq +1)$ be large and positive. A negative value indicates smaller predicted yields occurring with larger observed yields (and vice versa).

Current Measure of Modeled Yield Reliability
Defined by a Correlation Coefficient

One of the model characteristics to be evaluated is its ability to provide an accurate, current measure of modeled yield reliability.

Although a specific statistic was not discussed in the paper, Crop Yield Model Test and Evaluation Criteria, (Wilson, et. al., 1980), it was stated that:

"This 'reliability of the reliability' characteristic can be evaluated by comparing model generated reliability measures with subsequently determine deviation between modeled and 'true' yield."

For regression models, this suggests the use of a correlation coefficient between two variables generated for each test year. One variable is an indicator of a precision with which a prediction for the next year can be made, based on the model development base period and prediction year independent variable values. The other variable (obtained retrospectively) is an indicator of how close the predicted value for the next year actually is to the "true" value. The estimate of the standard error of a predicted value from the base period model as applied in the prediction year is used for the first value, \hat{s}_y , and the absolute value of the difference between the predicted and actual yield in the test year is used as the second variable, $|d|$. Since \hat{s}_y incorporates current-year weather as compared to long-term average, if the relations of yield to trend and weather specified in the model are valid the magnitude of \hat{s}_y should fluctuate in phase with $|d|$.

A non-parametric (Spearman) correlation coefficient, r , is employed since the assumption of bivariate normality cannot be made. A positive value of $r(-1 \leq r \leq +1)$ indicates agreement between \hat{s}_y and $|d|$, i.e., a

smaller (larger) value of \hat{s}_y is associated with a smaller (larger) value of $|d|$. An r value close to +1 is desirable since it indicates that a small standard error of prediction (and therefore a narrow confidence interval about the true predicted value) is associated with small discrepancies between predicted and actual yields. If this were the case, one would have confidence in using \hat{s}_y as an indicator of the accuracy of Y .

MODEL EVALUATION

Plots of actual and predicted yields for MN and ND state level models are presented in Figures 2 and 3. Results of the ten-year bootstrap tests on which these evaluations were based are presented in Appendix 1.

Indicators of yield Reliability Based on d Show Moderate Bias, Standard Deviations Ranging from 1.8 to 2.9 Q/Ha, and RMSE Ranging From 2.3 to 4.2 Q/Ha

The indicators of yield reliability based on deviations d ($d = \hat{Y} - Y$) at CRD, state, and region levels are given in Table 2. Root mean squared errors are presented in Figure 4.

CRD level biases for ND range from -3.3 to +1.1 Q/Ha, with all but one value negative. The biases for the MN CRD's are -0.9 and -3.1 Q/Ha. The Williams-type model seems to be biased overall at CRD level by about -1.4 Q/Ha.

Root Mean Square Errors (RMSE) for ND CRD's range from 2.3 to 4.4 Q/Ha and for MN from 3.1 to 4.2 Q/Ha. State level RMSE values were on the order of 3.0 Q/Ha overall.

Standard Deviation values ranged from 1.8 to 2.9 Q/Ha for ND CRD's with 2.9 Q/Ha for both MN CRD's. State and regional values ranged from 2.1 to 2.4 Q/Ha.

Examination of plots of observed and predicted yields at state level in Figures 1 and 3 indicates that in both ND and MN the Williams-type model

prediction seem to be biased by a consistent negative 2 Q/Ha in the years 1975-1979. This may indicate a weakness in the Williams-type model and is discussed in the CONCLUSIONS section.

Indicators of Yield Reliability Based on rd Show
that a Large Number of Cases have 50 Percent
or More of Test Years with | rd |
Greater than 10 Percent

The CRD, state and region values for the indicators of yield reliability based on relative difference | rd | are given in Table 3 and Figures 5, 6, and 7.

Eight of the nine ND CRD's and one of the two MN CRD's show 50 percent or more of the test years with | rd | greater than 10 percent. State and regional results show all six cases with 50 percent or more of the test years | rd | greater than 10 percent. These results would seem to indicate either a large natural variability in barley yields or a low level of model skills. Both are supported by the plots in Figures 2 and 3. If the model capabilities could be significantly improved in the years 1975-1979 the indicators of yield reliability would also be much improved.

For ND 1974 was the year with the largest relative difference in five of nine CRD's. In three CRD's 1973 was largest, and in one 1976 showed the largest difference. All 1974 cases represented an inability of the model to respond to a very low actual yield while the 1973 and 1976 cases represented inability to respond to a high actual yield. For MN 1976 and 1977 were the worst years for model performance.

Indicators of Yield Reliability Based on \hat{Y} and Y
Show Moderately Good Correspondence
Between the Direction of Change in Predicted
Yield Compared to Actual Yield

The predicted and actual yields at the state level are plotted in Figures 2 & 3. The predicted yields, actual yields, and differences for CRD level are listed in Appendix 1. The CRD, state, and region level

values for indicators of yield reliability based on actual and predicted yields are given Table 4 and Figures 8, 9 and 10.

Seven of the nine ND and both MN CRD's show a change of direction of predicted yields from the previous year corresponding to the actual change of direction more than 50% of the time. For state and regional models the response direction from the previous year is correct more than 50% of the time in two of six cases, and from the three year average in all six cases. These results indicate that the Williams-type model does moderately well in responding to changes of actual yield, particularly changes from a three-year base period.

Results for the correlation coefficient, r , between predicted and actual yields, representing correlation between fluctuations of predicted and actual yields from test period averages, appear fairly good. Of the eleven CRD's six show r greater than 0.55 (the level required for one-tailed statistical significance). The score for state and regional models is all six greater than 0.55. While the directional response capabilities of the model show some reliability, it should be kept in mind that r measures primarily correctness in direction of response. While response direction seems to be good, a glance at Figures 2 and 3, and at RMSE levels in Table 2, indicates that the Williams-type model leaves much to be desired in the correctness of magnitude in the responses.

Base Period Indicates More Precision Than
Independent Tests Can Confirm

Certain statistics generated from the regression analysis of the base period data are often used to provide some indication of expected yield reliability. However, these statistics only reflect how well the model describes the data used to generate the model, i.e., fit of the model, rather than how well the model can predict given new data. Therefore, it

is important to compare these indicators of fit of the model to the independent indicators of yield reliability discussed in the preceding sections. In this way, one can see how these base period indicators of fit of the model do or do not correspond to independent test indicators of yield reliability.

One indicator of yield reliability, the mean square error (MSE), is the sum of squared d values ($d = \hat{Y} - Y$) for the independent test years divided by the number of test years (Table 2). The direct analogue for the model development base period is the residual mean square. The residual mean square is obtained by first generating the usual least squares prediction equation using the base period years. The residual mean square is the sum of squared d values for these base period years divided by the appropriate degrees of freedom (number of base period years minus number of parameters estimated in fitting the model). Whereas one value of MSE is generated for each geographic area over the entire test period, a value of the residual mean square is generated for each period corresponding to an individual test year.

High, low, and average values of residual mean square for CRD and state models are given in Table 5, along with the mean square error over the test years for each. The MSE over the independent test years ranges from 2.0 to 8.7 times the corresponding average residual mean square error.

Another indicator of yield reliability is the correlation coefficient, r , between the observed and predicted yields for the independent test years (Table 4). It is desirable for r to be close to +1, even though it can be negative. The analogue for the model development base period is the square root of R^2 , the coefficient of multiple determination. The square root of R^2 , R ($0 \leq R \leq 1$), may be interpreted as the correlation between observed

and predicted values for the base period years. The low, high, and average values of R for each geographic area are given in Table 6, along with the Pearson correlation coefficient values from Table 4.

Average correlation coefficients over the base period (model development years) range from .93 to .97, indicating the model is doing a very good job of fitting the development data. The correlation coefficients over the independent test years range from a low of .47 and to a high of .85. The r value over the independent test years is generally only about two-thirds the average R for the model development years. Clearly the Williams type model does not respond as well in a predictive mode as in a fitting mode. The values of R for model development years do not provide an effective indication of the predictive abilities of the model.

Model is Reasonably Objective

The Williams-type model is redeveloped (i.e., values of coefficients are re-derived) for each test year, based on available years prior to it. Once the proper terms have been selected and fixed, development and application of the model is quite objective. Some subjectivity is required for initially selecting the "most significant" terms, in specifying trend, particularly break points, in specifying textural and topographic data, and in choice of development years.

Model Considers Known Scientific Relationships on a Broad Scale

Large-area crop yields are known to be related to weather over the growing season, to pre-season stored soil moisture, and to a variety of other weather and agronomic factors. The details of the mathematical relationships that describe these physical and biological relationships are far from established. Even the proper set of variables is open to question because there are only a few readily available observables and the

variables formed from these tend to be highly interrelated. Large-area relationships are further confused by geographical variations in the observables that may or may not be important for any given situation.

In light of these problems a practical approach was used consisting of statistical regression of observed yields to variables based on monthly weather data pooled to regional level. Technological impacts were represented as a function of historical years (trend), and a policy of refitting for each predictive year based on all available prior years was followed. Thus the Williams-type model is susceptible to criticism in regard to agreement with scientific knowledge in many respects. A few of the more important are noted below.

Selection of model terms is by stepwise regression. This guarantees only the set of terms "best" by some statistical criterion. Physical or biological significance is not ensured. It seems unlikely that the wide variety of "significant" terms represented in Appendix 2 for different models has a great deal of physical meaning. Of particular note are the textural and topographic terms found by Williams to be very important in his large Canadian regions. One would expect these terms to show up in the NDREM region (ND CRD's 10,20,40,50, 70,80,90) since there is a great deal of variation over these regions, whereas the MNRR region (MN CRD's 10,40, ND CRD's 30,60) is quite homogeneous. Appendix 2 shows that TX and TOP appear only in the MNRR model and TXDS appears in only the ND state model. The terms appear to be functioning mainly as artificial variables that happen in one case to be more significant than another. Little or no physical significance can be attached to them. The selection criteria in general have not been documented.

Trends in technology and cropping practices are handled in the Williams-type model by representing them as piecewise linear and/or quadratic functions of time. This glosses over the known qualitative relationship of yield to variety improvement, fertilizer use, etc., but represents a practical way of treating the situation where it is unclear which effects are most important and where information is limited. Following Williams' original approach, single trends were specified for the pooled sets, clearly an oversimplification. An assumption of pooling is that the inherent fertility of the pooled areas is the same (common intercept) for equivalent weather and soil types. Inclusion of textural and topographic variables, parameters known to have a real effect, were intended to modify trend in different parts of the region. However, in view of the way they actually entered the models, they did not function in this manner.

The Williams type model takes no explicit account of pests, disease, or other episodic events.

Model is Adequate Only for the Region
in Which It Was Developed

By its nature the Williams-type model can be applied with any degree of reliability only in the region for which it was developed. The model is probably not extendable even to apparently similar regions. The model can, however, be readily applied to any region for which a reasonable lengthy record of yield, soil and weather observations exist.

The Williams-type model may have an advantage over regression-type models developed at the smallest available regions (here CRD's) in the case of short data records. Pooling provides a larger data set for the determination of significant terms and coefficients while still giving yield predictions at the small region level.

Model is Timely Enough for
Intended Applications

A yield model for a new year can be built as soon as the reliable yield and weather variable figures from the past year are available, in the U.S. generally a few months after harvest. Yield predictions during an application year can be made shortly after the end of each month.

Model is Not Costly

Data to develop and run the Williams-type model are readily available at low cost. The multiple regressions needed to compute the meteorological and agronomic variables and develop models can be run on any modest size computer. Routines are available in most computer libraries.

Model is Simple

The development and application of the Williams-type model are straightforward. The only points where judgement is required are in selection of significant terms and specification of trend, selection of soils variably, and specifying the capacity of the soil moisture budget.

Model Has Poor Current Measure of
Modeled Yield Reliability

The CRD, state, and region values of the correlation coefficient between the estimate of the standard error of the predicted yield values and the absolute differences between predicted and actual yield are presented in Table 7 and Figure 11. The results are very poor. In eight of thirteen cases the correlation is negative. State models show negative r . It is clear from the Spearman correlation coefficient that the base period predicted accuracy and actual test year accuracy are not in close agreement and thus the model does not give a useful current measure of modeled yield reliability.

CONCLUSIONS

The Williams-type model represents an approach involving pooling of CRD level data to derive a model for CRD's within that region. This provides a good deal more data for a regression at the regional level but may gloss over any CRD to CRD differences. Therefore the approach is a compromise in principle and its validity in practice can only be evaluated by testing. The data bases consist of observed yields, soil characteristics, and monthly mean temperature and total precipitation. Indicators of yield reliability obtained from bootstrap testing are used as a basis for evaluating model performance. Over the set of ten test years the model is reasonably reliable on-average. Biases are not large but seem to be slightly on the negative side. Root mean square errors over the ten test years are in the range of 3 Q/Ha, somewhat larger than one would prefer but appear reasonable for the intended AgRISTARS large area applications. The Williams-type model does not consistently predict high and low yields very reliably, and for any given year the actual error may be appreciably larger than the RMSE value across the 10 years. The model does not give a good current measure of yield reliability. However, it is objective, adequate for intended purposes, timely, simple, not costly, and makes a practical attempt at incorporating scientific knowledge.

Many general areas of needed improvement could be cited. The most obvious specific area is to determine why the Williams-type model seems to be consistently biased low in ND and/or from 1975 thru 1979. Elimination of this problem would appreciably improve RMSE and probably other indicators as well. A fit made with the TREND2SQ term removed, leaving linear trend segments 1931-1961 and 1962-1979, gave predicted yields coinciding almost exactly with actual yields in 1975-1979 but with much poorer performance in 1970-1974. Across the ten year test period the RMSE for this

alternative model fit was slightly worse than that for the original Williams-type model. Clearly, the fix is not such a simple adjustment.

Another area that should be investigated is why the textural and topographic variables enter in one case and not another. There is, for this reason, some question as to what these two variables really contribute.

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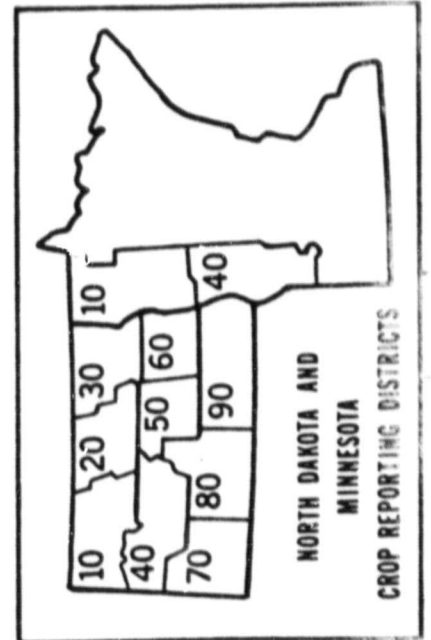
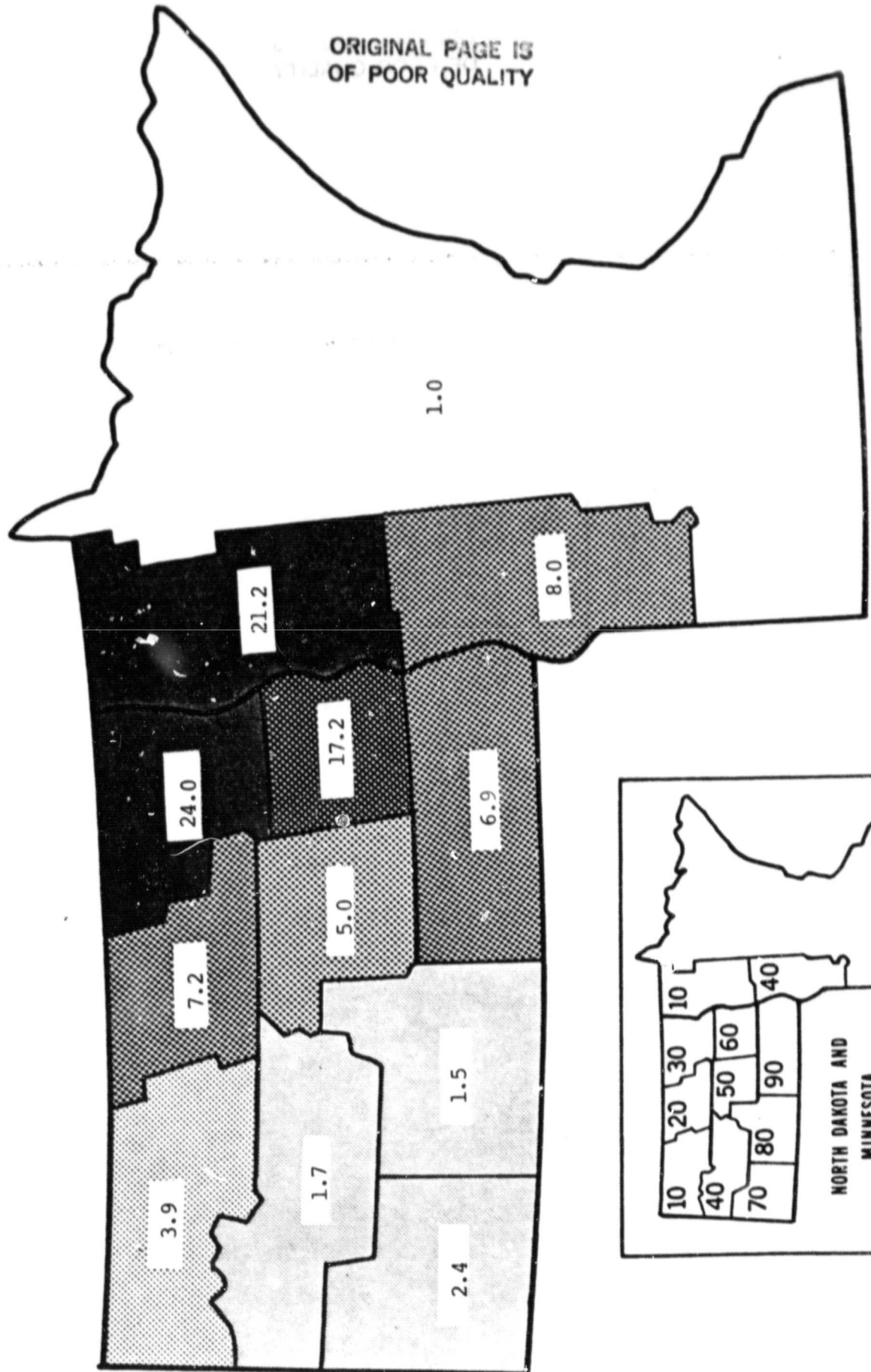
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TABLE 1
AVERAGE PRODUCTION AND YIELD
FOR TEST YEARS 1970-79

BARLEY
NORTH DAKOTA AND MINNESOTA

STATE	CRD	PRODUCTION (1,000)		PERCENT OF		YIELD	
		QUINTALS	BUSHEL	STATE	REGION	QNTL/HA	BU/ACRE
N. DAKOTA	10	1,081	4,964	5.7	3.9	19.4	36.1
	20	1,964	9,023	10.3	7.2	18.9	35.2
	30	6,559	30,126	34.3	24.0	21.9	40.8
	40	473	2,171	2.5	1.7	19.4	36.1
	50	1,374	6,309	7.2	5.0	18.8	34.9
	60	4,700	21,588	24.6	17.2	23.3	43.2
	70	647	2,972	3.4	2.4	18.9	35.1
	80	423	1,943	2.2	1.5	16.0	29.7
	90	1,885	8,659	9.9	6.9	20.0	37.2
STATE		19,106	87,754		69.8	21.0	39.1
MINNESOTA	10	5,801	26,646	70.1	21.2	24.9	46.2
	20	43	196	0.5	0.2	18.5	34.4
	30	5	21	0.1	0.0	19.7	36.5
	40	2,203	10,119	26.6	8.0	22.3	41.5
	50	77	353	0.9	0.3	20.8	38.8
	60	20	92	0.2	0.1	20.5	38.2
	70	51	235	0.6	0.2	20.9	38.9
	80	17	80	0.2	0.1	23.7	44.0
	90	55	252	0.7	0.2	24.7	46.0
STATE		8,272	37,994		30.2	24.0	44.6
REGION		27,378	125,748			21.9	40.6

Figure 1. Production of barley by CRD (1970-1979 average) as a percent of the regional total. Darker shades indicate CRDs with higher production.



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Figure 2
Actual and Predicted Yields

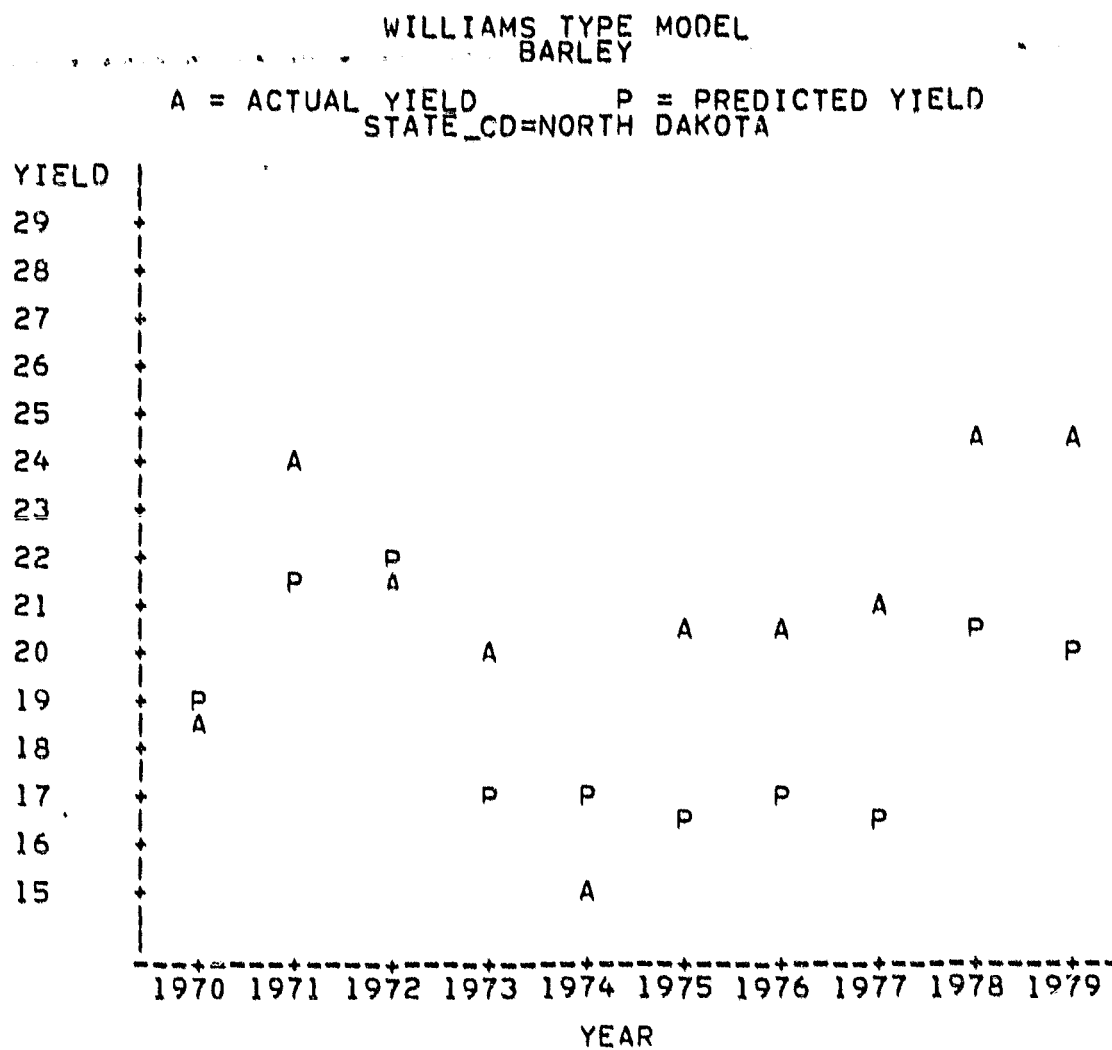
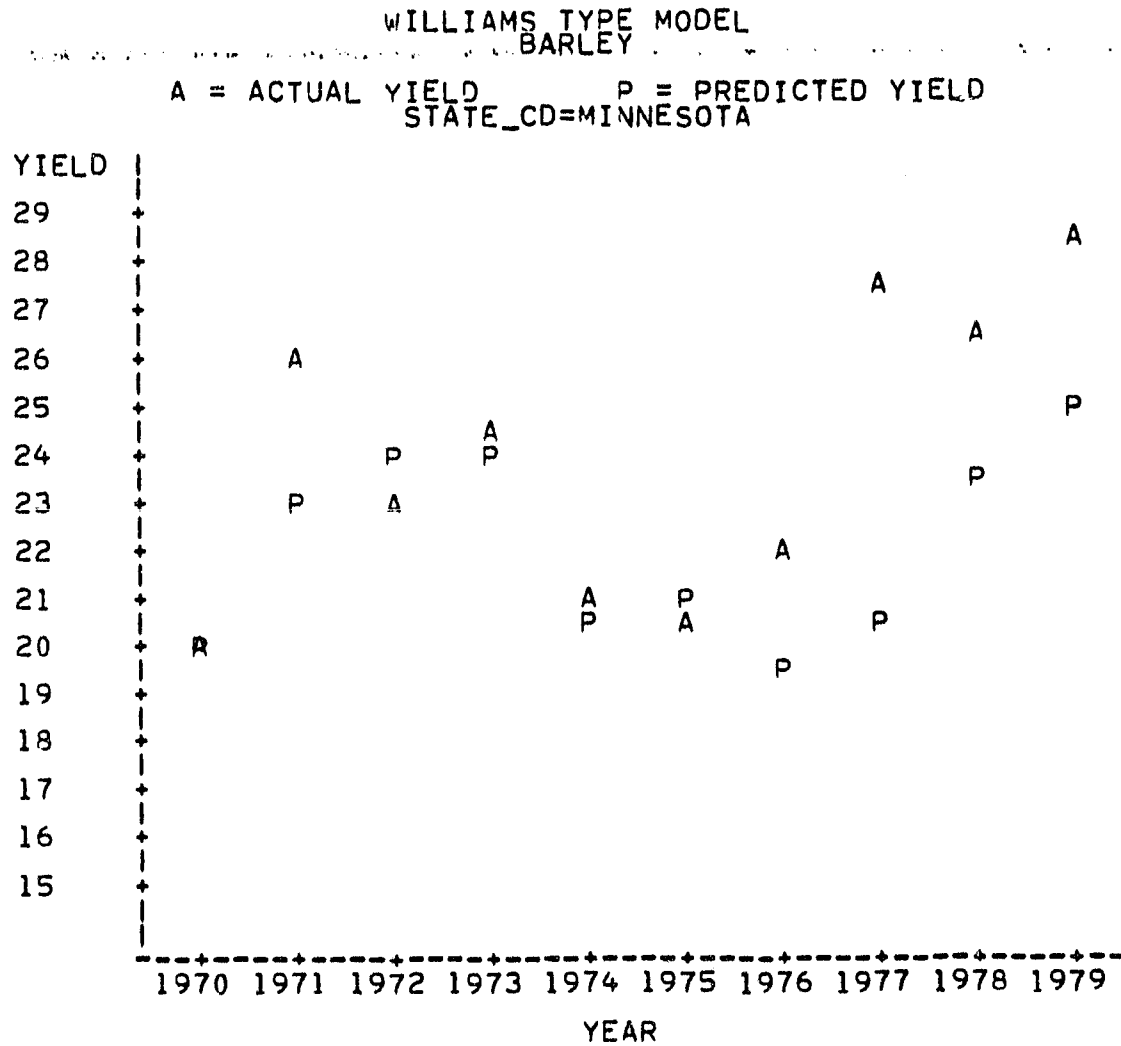


Figure 3
Actual and Predicted Yields

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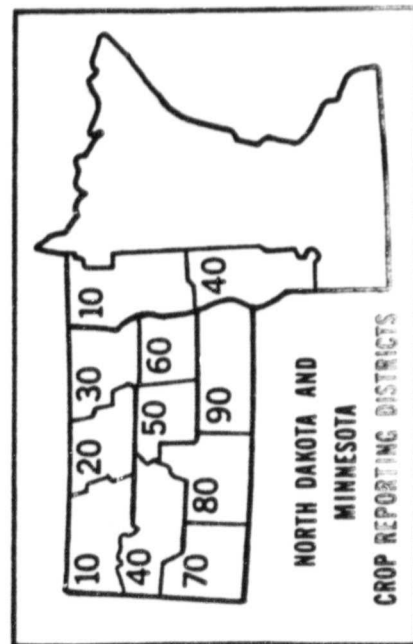
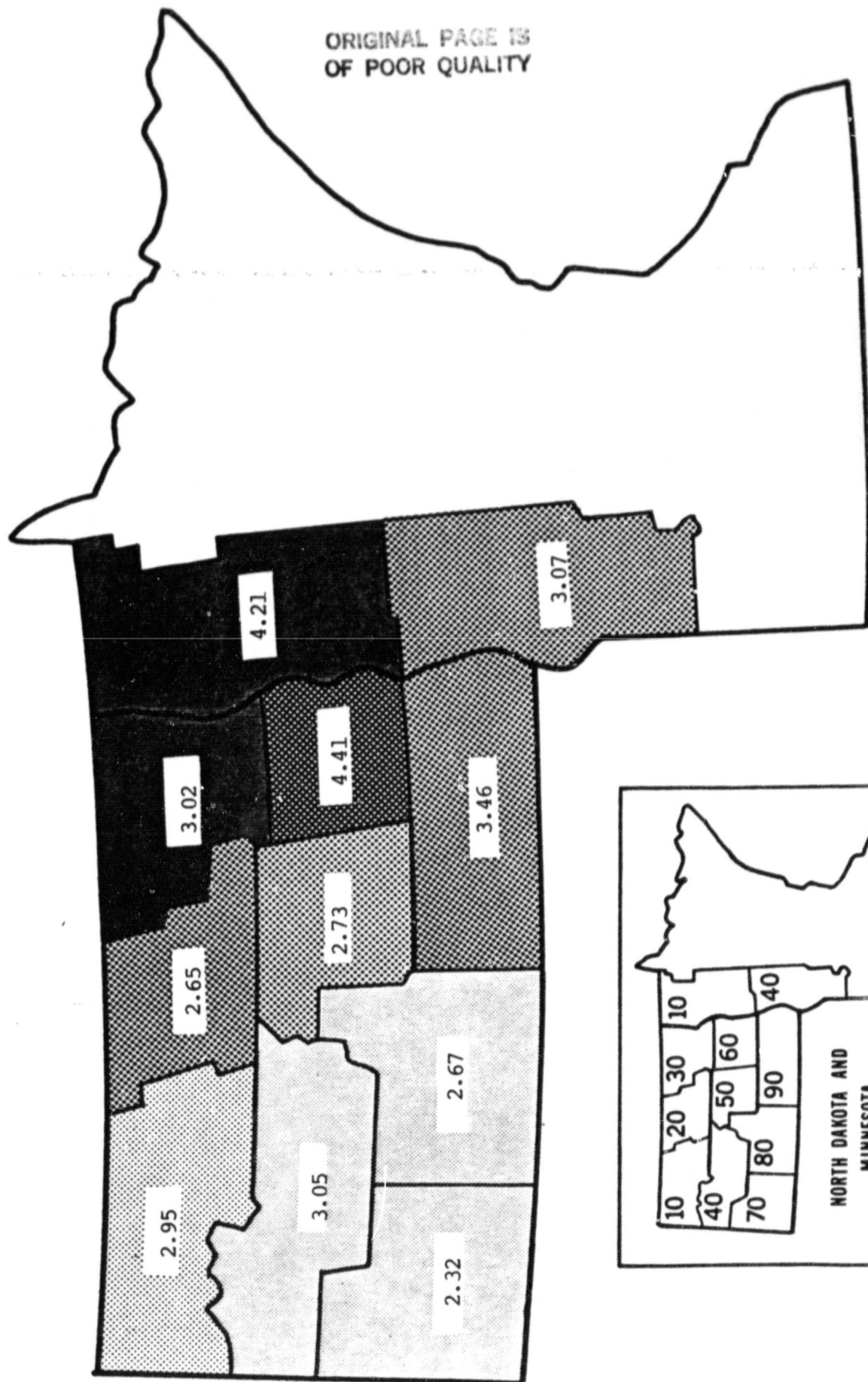
TABLE 2
INDICATORS OF YIELD RELIABILITY
BASED ON D = PREDICTED - ACTUAL YIELD

WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

MSE, VAR, B-SQR (QUINTALS/HECTARE SQUARED)
RMSE, SD, BIAS (QUINTALS/HECTARE)
RRMSE, RSD, RB (PERCENT OF AVERAGE YIELD)

STATE	CRD	MSE	RMSE	RRMSE	VAR	SD	RSD	B-SQR	BIAS	RB
N. DAKOTA	10	8.73	2.95	15.2	7.61	2.76	15.0	1.12	-1.06	-5.5
	20	7.04	2.65	14.0	6.84	2.62	14.2	0.20	-0.45	-2.4
	30	9.11	3.02	13.8	7.78	2.79	13.4	1.32	-1.15	-5.2
	40	9.32	3.05	15.7	7.36	2.71	15.1	1.96	-1.40	-7.2
	50	7.45	2.73	14.5	5.81	2.41	13.8	1.64	-1.28	-6.8
	60	19.46	4.41	19.0	8.51	2.92	14.6	10.96	-3.31	-14.2
	70	5.38	2.32	12.3	5.04	2.24	12.3	0.34	-0.58	-3.1
	80	7.12	2.67	16.7	5.98	2.44	14.3	1.14	-1.07	-6.7
	90	11.99	3.46	17.3	3.29	1.81	10.6	8.70	-2.95	-14.7
STATE MODEL		10.92	3.30	15.7	5.58	2.36	12.6	5.34	-2.31	-11.0
CRDS AGGR.		7.83	2.80	13.3	4.77	2.18	11.3	3.06	-1.75	-8.3
MINNESOTA	10	17.71	4.21	16.9	8.35	2.89	13.3	9.36	-3.06	-12.3
	40	9.41	3.07	13.8	8.66	2.94	13.7	0.76	-0.87	-3.9
STATE MODEL		8.81	2.97	12.4	5.20	2.28	10.3	3.61	-1.90	-7.9
CRDS AGGR.		11.15	3.34	13.9	5.48	2.34	10.8	5.66	-2.38	-9.9
REGION										
CRDS AGGR.		8.15	2.85	13.1	4.35	2.09	10.5	3.80	-1.95	-8.9
STATES AGGR.		9.42	3.07	14.0	4.58	2.14	10.9	4.84	-2.20	-10.1

Figure 4. Root mean square error (RMSE) for barley in quintals per hectare based on test year 1970-1979. Darker shades indicate CRDs with higher production.

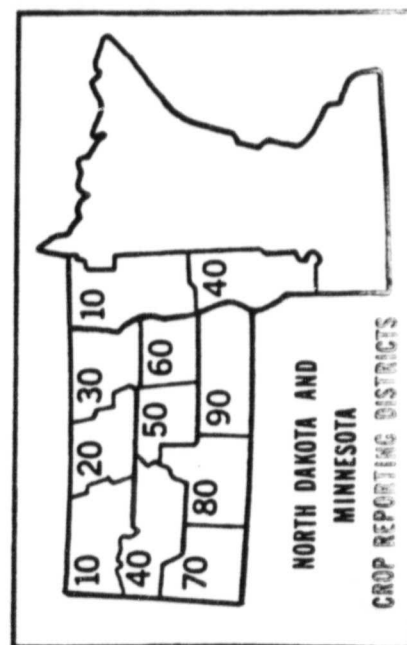
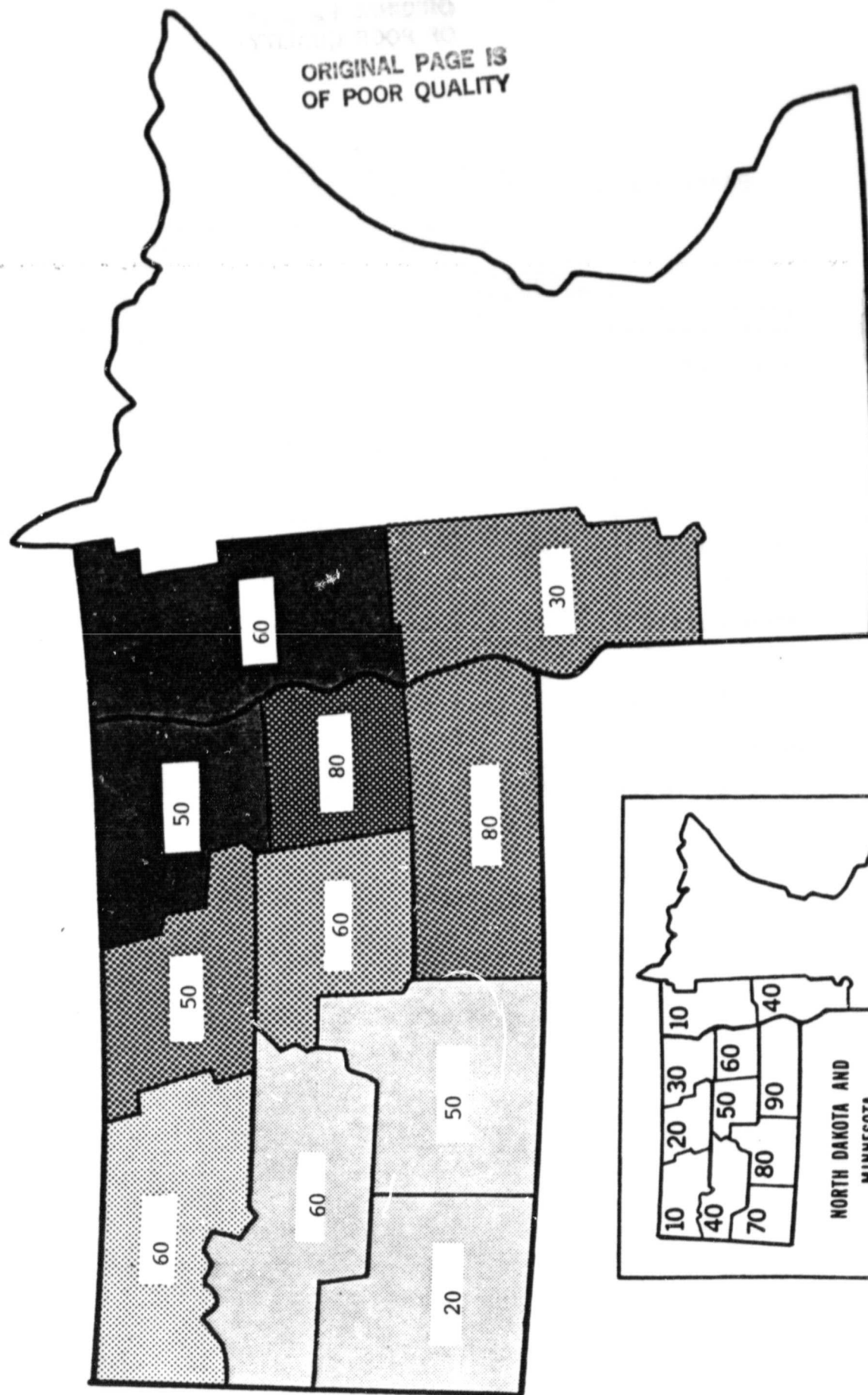


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TABLE 3
INDICATORS OF YIELD RELIABILITY
BASED ON $RD = 100 * ((\text{PREDICTED} - \text{ACTUAL YIELD}) / \text{ACTUAL YIELD})$
WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

STATE	CRD	PERCENT OF YEARS IRDI > 10%	LARGEST IRDI RD (YEAR)	NEXT LARGEST	SMALLEST IRDI	RANGE IRDI
N. DAKOTA	10	60	-28.4 (1973)	26.8	1.8	26.6
	20	50	48.4 (1974)	-18.3	-0.6	47.8
	30	50	31.1 (1974)	-22.5	0.5	30.6
	40	60	37.6 (1974)	-26.9	-1.3	36.4
	50	60	40.7 (1974)	-20.8	-1.0	39.7
	60	80	27.6 (1976)	-23.1	0.0	27.6
	70	20	-24.9 (1973)	-17.9	-1.0	23.9
	80	50	38.6 (1974)	36.2	-1.7	36.9
	90	80	-38.5 (1973)	-18.8	-2.3	36.2
STATE MODEL		80	-21.4 (1977)	-20.1	2.8	18.6
CRDS AGGR.		70	19.2 (1974)	-19.1	1.1	18.1
MINNESOTA	10	60	-27.7 (1977)	-26.2	-1.6	26.1
	40	30	33.1 (1976)	-24.2	0.0	33.1
STATE MODEL		50	-24.8 (1977)	-13.0	0.0	24.8
CRDS AGGR.		50	-26.9 (1977)	-14.6	-1.3	25.6
REGION						
CRDS AGGR.		70	-17.9 (1977)	-17.7	2.7	15.2
STATES AGGR.		70	-22.7 (1977)	-16.2	2.7	20.0

Figure 5. Percent of test years (1970-1979) the absolute value of relative difference is greater than ten percent for barley. Darker shades indicate CRDs with higher production.



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Figure 6. Largest absolute value of the relative difference for barley during the test years 1970-1979. Darker shades indicate CRDs with higher production.

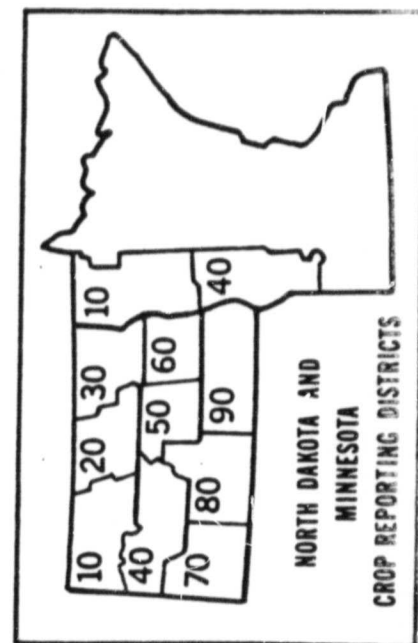
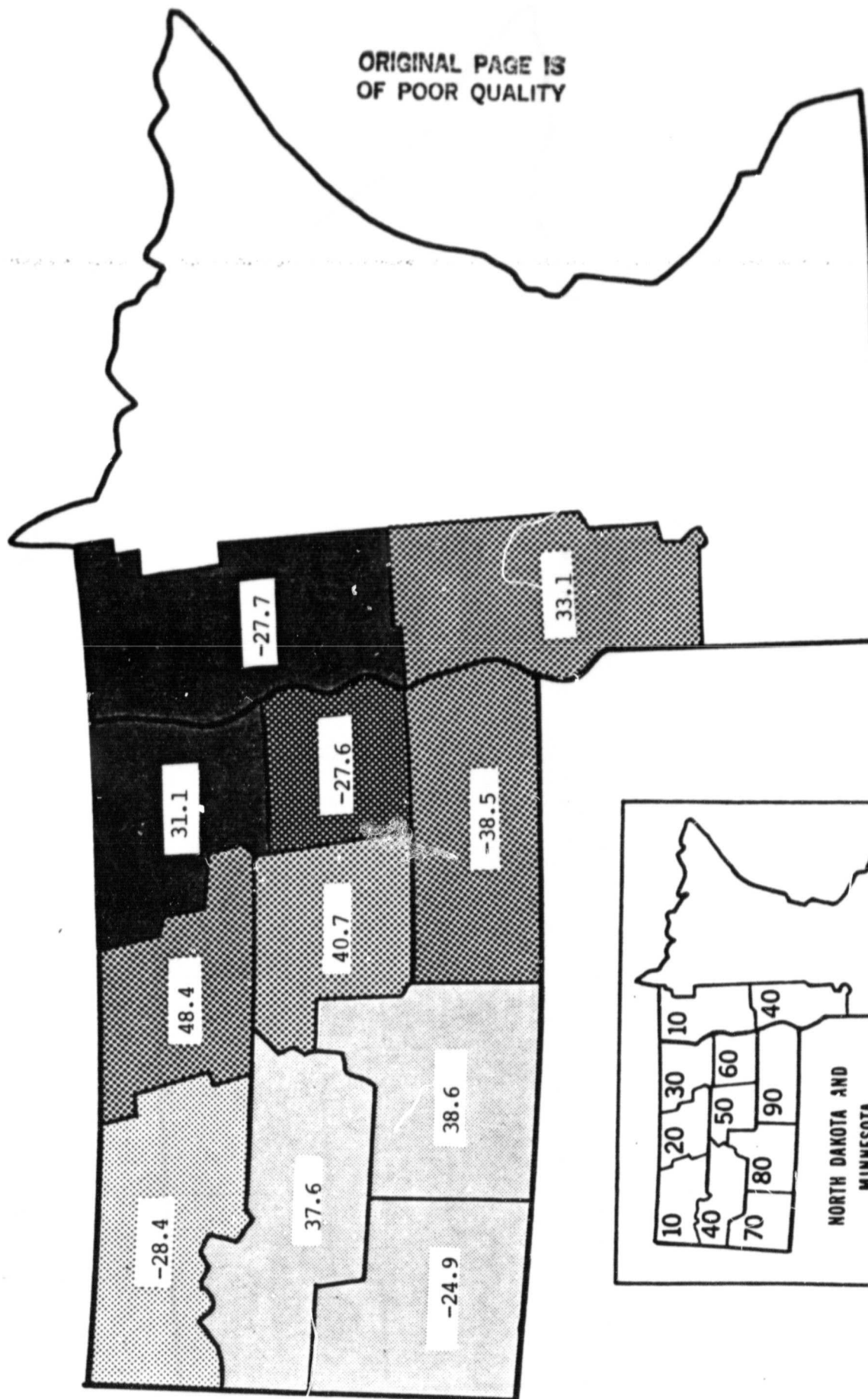
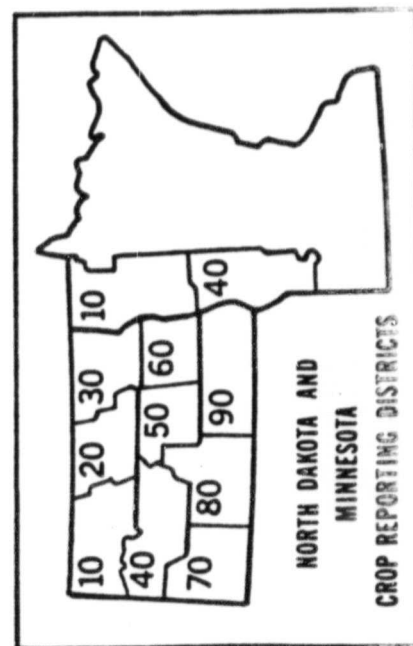
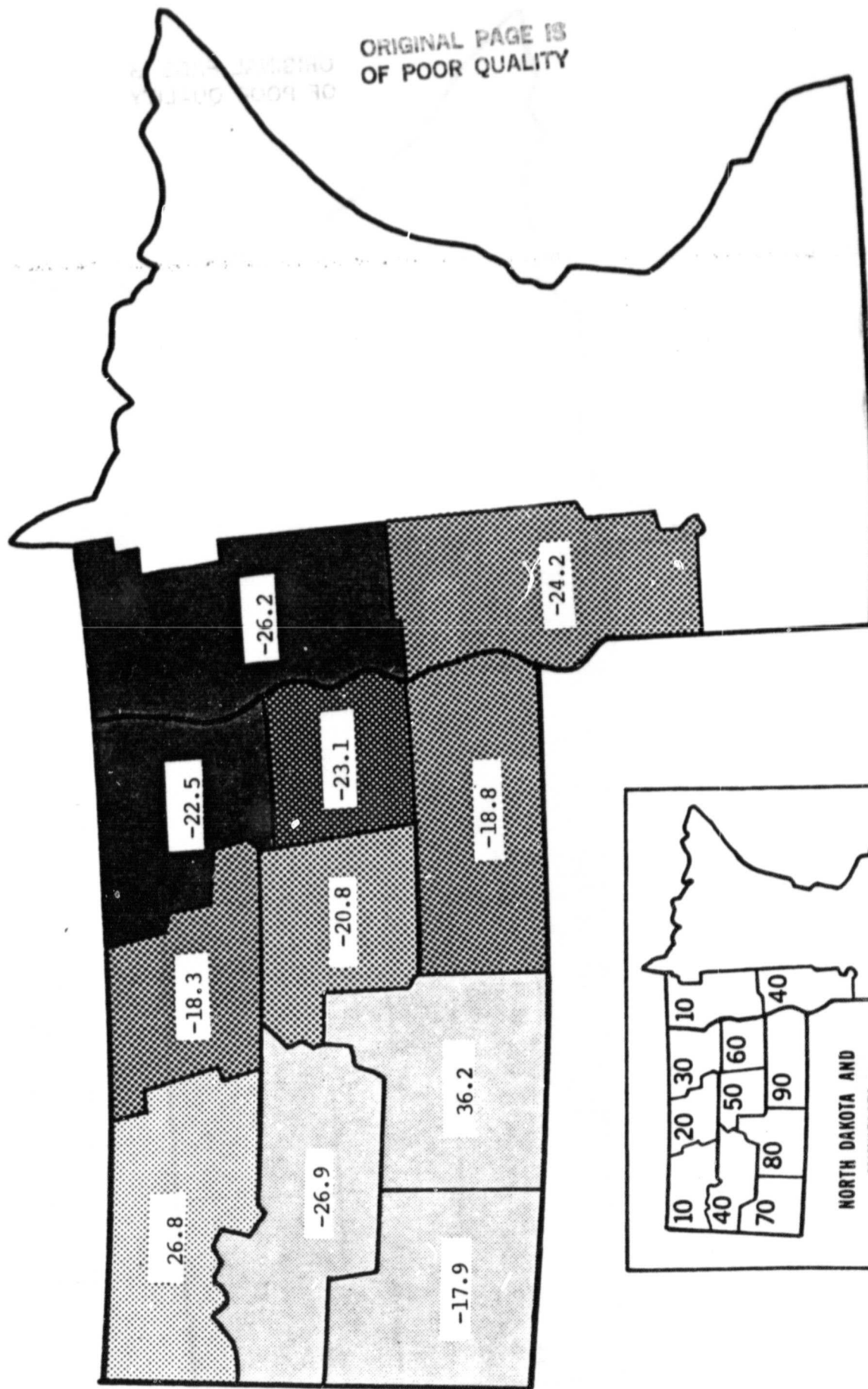


Figure 7. Next largest absolute value of the relative difference for barley during the test years 1970-1979. Darker shades indicate CRDs with higher production.



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TABLE 4
INDICATORS OF YIELD RELIABILITY
BASED ON ACTUAL AND PREDICTED YIELDS

WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

STATE	CRD	PERCENT OF YEARS DIRECTION OF CHANGE IS CORRECT		PEARSON CORR. COEF.
		FROM PREVIOUS YEAR	FROM BASE PERIOD	
N. DAKOTA	10	67	71	0.55
	20	67	57	0.47
	30	67	71	0.51
	40	78	86	0.69
	50	67	71	0.73
	60	56	71	0.63
	70	44	57	0.52
	80	67	71	0.69
	90	44	100	0.85
STATE MODEL CRDS AGGR.		33	57	0.58
		44	71	0.65
MINNESOTA	10	89	71	0.54
	40	89	86	0.80
STATE MODEL CRDS AGGR.		44	86	0.63
		67	71	0.62
REGION MODEL CRDS AGGR.		44	71	0.68
	STATES AGGR.	67	57	0.64

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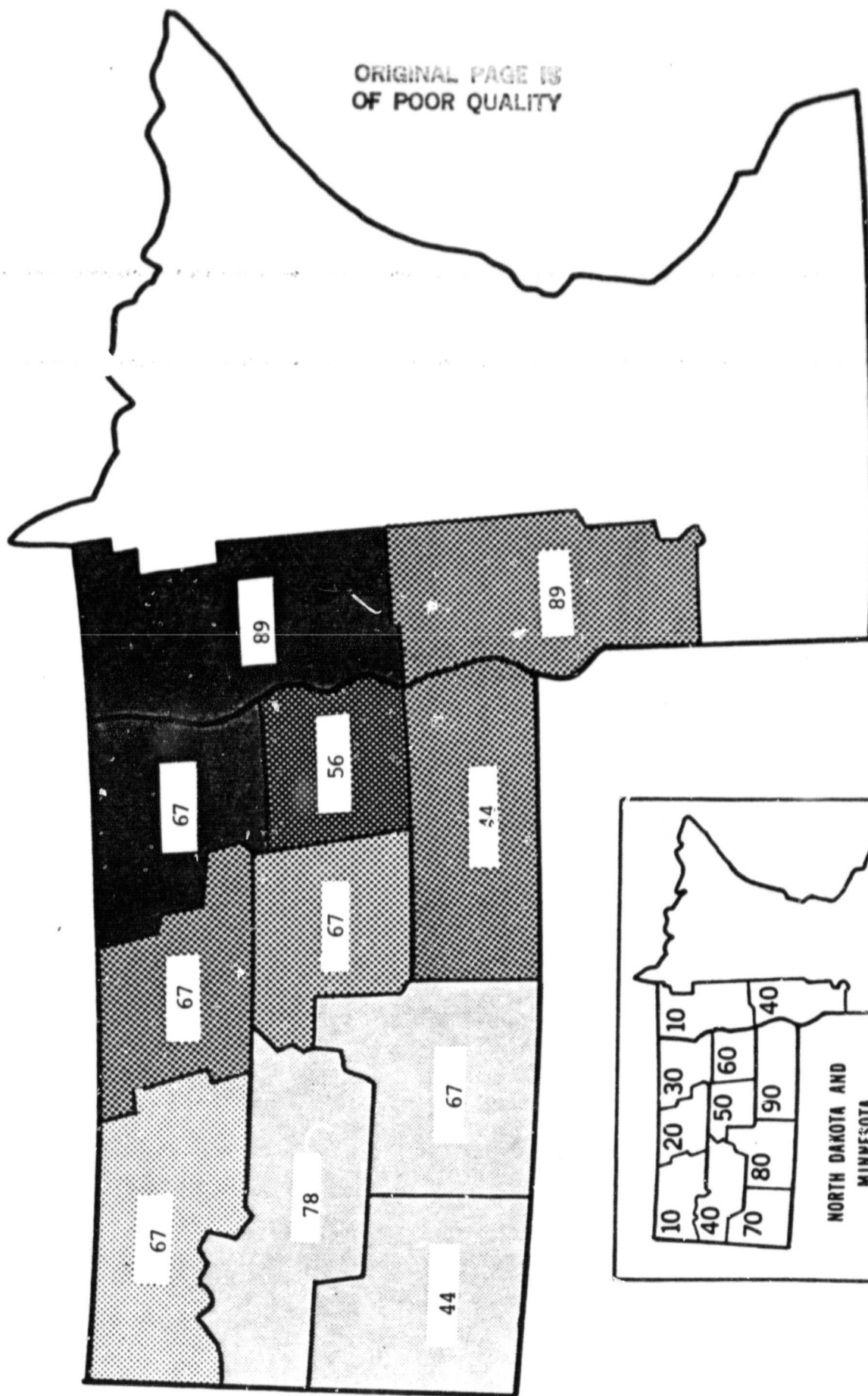


Figure 9. Percent of test years (1970-1979) the direction of change in predicted yield from the previous three year average agrees with the direction of change in actual barley yield. Darker shades indicate CRO's with higher production.

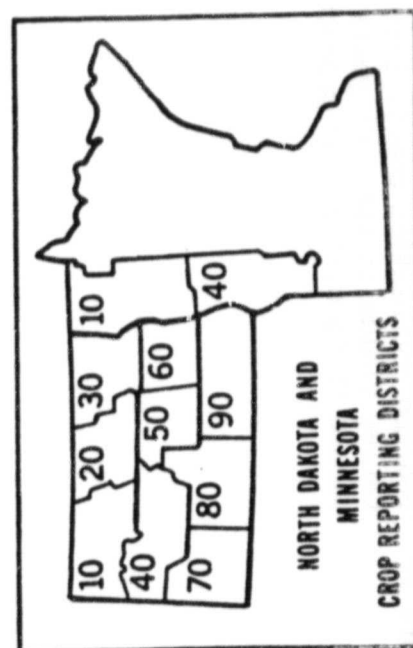
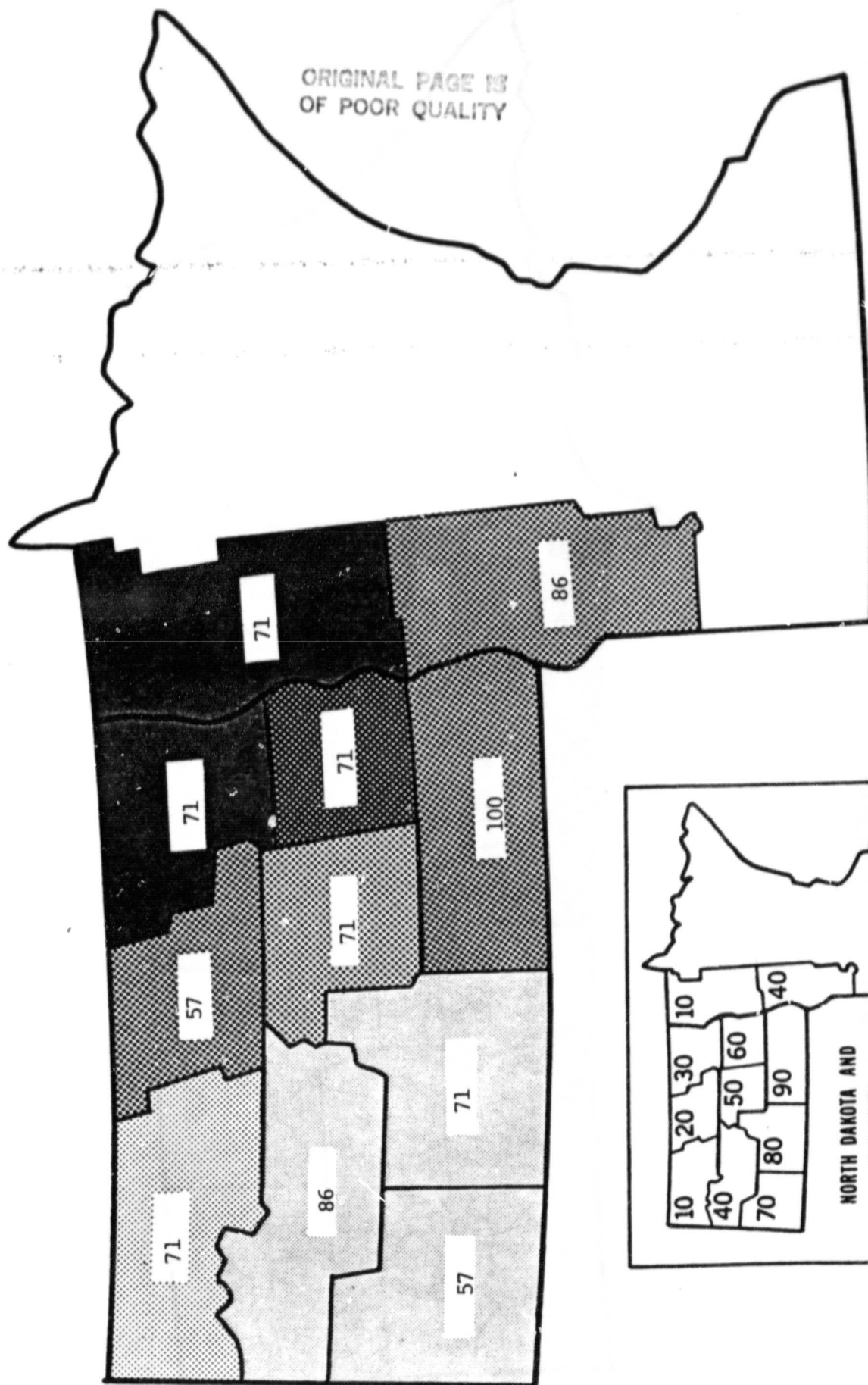
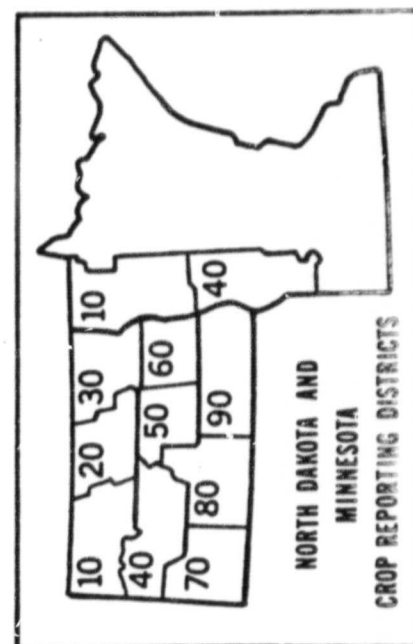
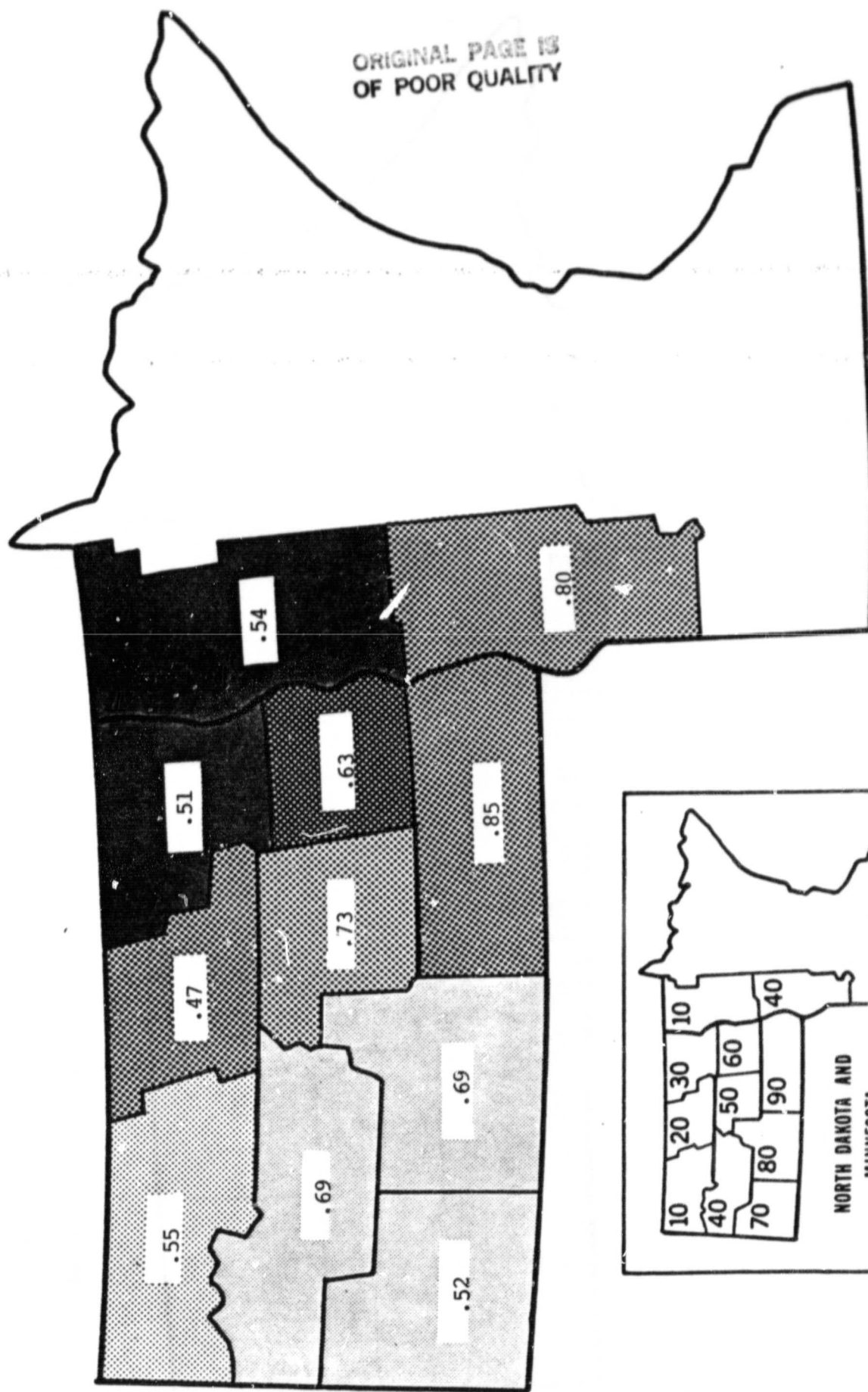


Figure 10. Pearson correlation coefficient between actual and predicted barley yields in the test years 1970-1979. Darker shades indicate CRD's with higher production.



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TABLE 5
RESIDUAL MEAN SQUARE AS AN
INDICATOR OF THE FIT OF THE MODEL
BASED ON THE MODEL DEVELOPMENT BASE PERIOD

WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

STATE	CRD	BASE PERIOD RESIDUAL MEAN SQUARE			INDEPENDENT TEST MSE
		LOW	HIGH	AVERAGE	
N.DAKOTA	10	3.20	3.75	3.51	8.73
	20	3.20	3.75	3.51	7.04
	30	3.45	4.81	3.84	9.11
	40	3.20	3.75	3.51	9.32
	50	3.20	3.75	3.51	7.45
	60	3.45	4.81	3.84	19.46
	70	3.20	3.75	3.51	5.38
	80	3.20	3.75	3.51	7.12
	90	3.20	3.75	3.51	11.99
STATE MODEL		1.25	2.21	1.52	10.92
MINNESOTA	10	3.45	4.81	3.84	17.71
	40	3.45	4.81	3.84	9.41
STATE MODEL		3.17	3.85	3.47	8.81

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TABLE 6
CORRELATION BETWEEN OBSERVED AND PREDICTED YIELDS AS AN
INDICATOR OF THE FIT OF THE MODEL
BASED ON THE MODEL DEVELOPMENT BASE PERIOD

WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

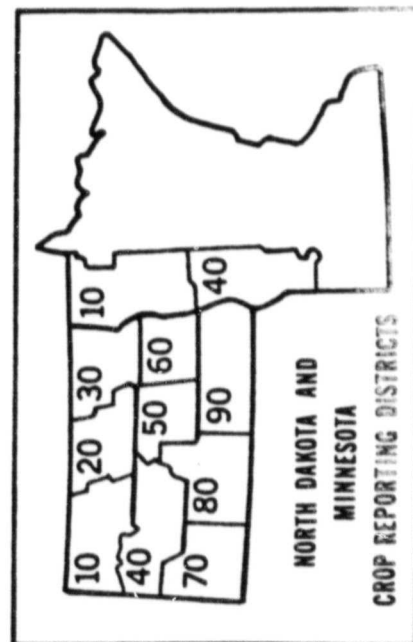
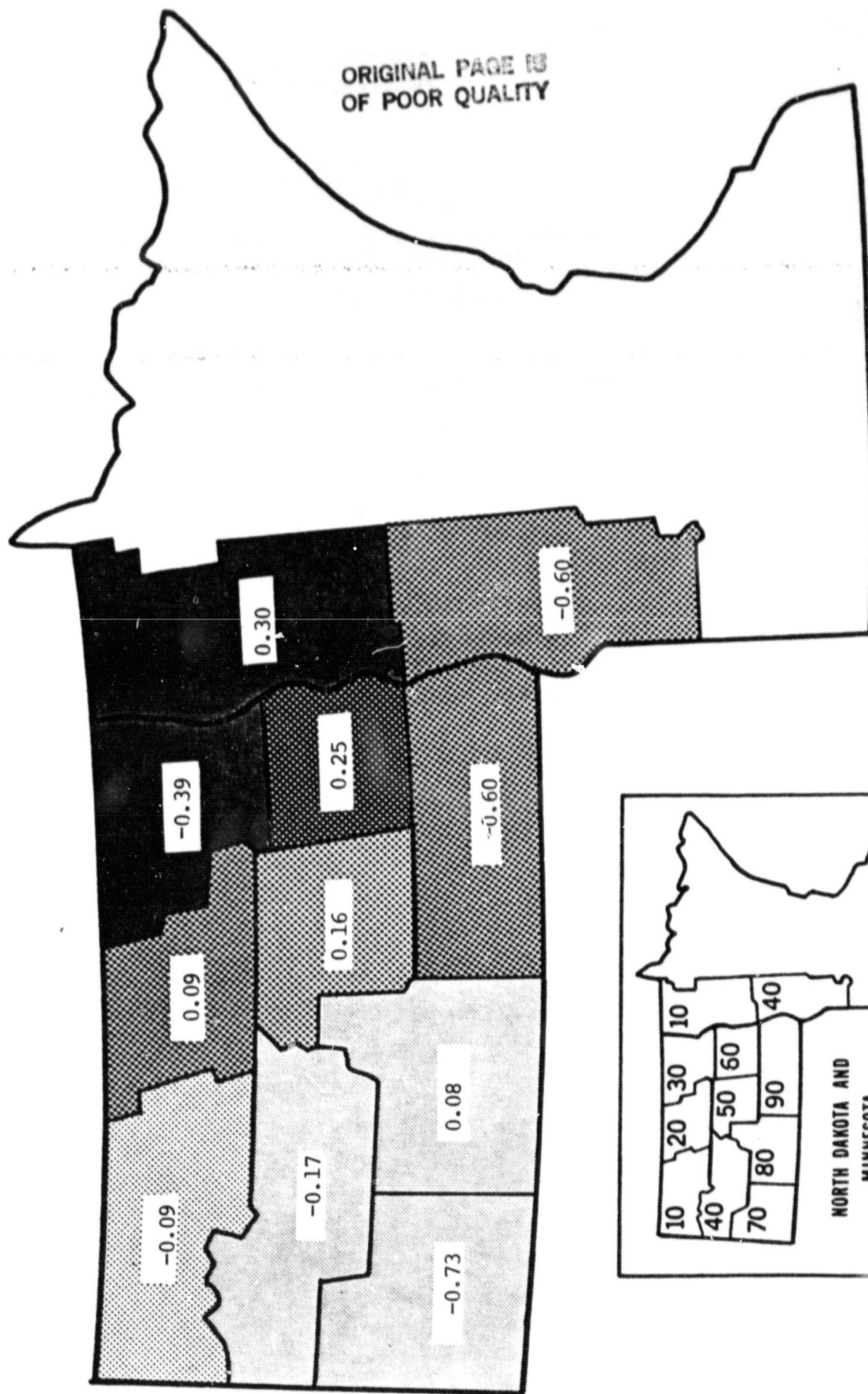
TEST STATE	CRD	BASE PERIOD CORRELATION COEF.			INDEPENDENT CORR. COEF.
		LOW	HIGH	AVERAGE	
N.DAKOTA	10	0.93	0.94	0.93	0.55
	20	0.93	0.94	0.93	0.47
	30	0.92	0.94	0.93	0.51
	40	0.93	0.94	0.93	0.69
	50	0.93	0.94	0.93	0.73
	60	0.92	0.94	0.93	0.63
	70	0.93	0.94	0.93	0.52
	80	0.93	0.94	0.93	0.69
	90	0.93	0.94	0.93	0.85
STATE MODEL		0.97	0.98	0.97	0.58
MINNESOTA	10	0.92	0.94	0.93	0.54
	40	0.92	0.94	0.93	0.80
STATE MODEL		0.91	0.94	0.93	0.63

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TABLE 7
CURRENT INDICATION OF
MODELED YIELD RELIABILITY
AGREEMENT BETWEEN BASE PERIOD PREDICTED
AND TEST YEAR ACTUAL ACCURACY
WILLIAMS TYPE MODEL - BARLEY
NORTH DAKOTA AND MINNESOTA

STATE	CRD	SPEARMAN CORRELATION COEF.
N.DAKOTA	10	-0.09
	20	0.09
	30	-0.39
	40	-0.17
	50	0.16
	60	0.25
	70	-0.73
	80	0.08
	90	-0.60
STATE MODEL		-0.20
MINNESOTA	10	0.30
	40	-0.60
STATE MODEL		-0.50

Figure 11. Spearman correlation coefficient between the estimate of the standard error of a predicted value from the base period model and the absolute value of the difference between the predicted and actual barley yield in the test years (1970-1979). Darker shades indicate CRD's with higher production.



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APPENDIX 1
BOOTSTRAP TEST RESULTS
FOR BARLEY YIELDS IN
NORTH DAKOTA AND MINNESOTA
USING A WILLIAMS TYPE MODEL

STATE	CRD	YEAR	YIELD (Q/H) ACTUAL	PRED.	D	RD	S.E. PRED.
N.DAKOTA	10	1970	20.1	17.6	-2.5	-12.4	2.02
		1971	20.9	20.0	-0.9	-4.3	2.00
		1972	21.0	24.0	3.0	14.3	2.08
		1973	22.5	16.1	-6.4	-28.4	2.02
		1974	13.8	17.5	3.7	26.8	2.12
		1975	16.6	16.9	0.3	1.8	2.11
		1976	19.0	17.8	-1.2	-6.3	2.15
		1977	18.6	16.2	-2.4	-12.9	2.16
		1978	25.0	22.4	-2.6	-10.4	2.08
		1979	16.9	15.3	-1.6	-9.5	2.12
	20	1970	18.6	17.4	-1.2	-6.5	2.02
		1971	21.4	20.8	-0.6	-2.8	2.00
		1972	20.6	23.1	2.5	12.1	2.03
		1973	20.4	18.1	-2.3	-11.3	2.01
		1974	12.2	18.1	5.9	48.4	2.15
		1975	17.7	16.6	-1.1	-6.2	2.10
		1976	19.8	17.6	-2.2	-11.1	2.16
		1977	16.4	16.3	-0.1	-0.6	2.19
		1978	22.5	20.7	-1.8	-8.0	2.07
		1979	19.7	16.1	-3.6	-18.3	2.12
	30	1970	19.5	20.2	0.7	3.6	2.39
		1971	24.5	23.6	-0.9	-3.7	2.27
		1972	21.9	23.2	1.3	5.9	2.15
		1973	20.1	20.2	0.1	0.5	2.13
		1974	14.8	19.4	4.6	31.1	2.16
		1975	22.7	17.6	-5.1	-22.5	2.09
		1976	22.3	18.2	-4.1	-18.4	2.11
		1977	21.8	19.0	-2.8	-12.8	2.24
		1978	24.4	22.8	-1.6	-6.6	2.33
		1979	27.2	23.5	-3.7	-13.6	2.37
	40	1970	17.1	17.9	0.8	4.7	2.03
		1971	21.5	18.5	-3.0	-14.0	2.09
		1972	23.9	23.6	-0.3	-1.3	2.02
		1973	20.8	15.2	-5.6	-26.9	2.02
		1974	11.7	16.1	4.4	37.6	2.14
		1975	17.4	17.1	-0.3	-1.7	2.11
		1976	19.9	16.9	-3.0	-15.1	2.10
		1977	16.7	16.4	-0.3	-1.8	2.16
		1978	25.5	21.4	-4.1	-16.1	2.10
		1979	19.6	17.0	-2.6	-13.3	2.11

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APPENDIX 1
BOOTSTRAP TEST RESULTS
FOR BARLEY YIELDS IN
NORTH DAKOTA AND MINNESOTA
USING A WILLIAMS TYPE MODEL

STATE	CRD	YEAR	YIELD ACTUAL	(Q/H) PRED.	D	RD	S.E. PRED.
N.DAKOTA	50	1970	17.7	16.3	-1.4	-7.9	2.01
		1971	24.5	20.5	-4.0	-16.3	1.99
		1972	20.4	20.2	-0.2	-1.0	2.00
		1973	14.5	12.9	-1.6	-11.0	2.05
		1974	12.3	17.3	5.0	40.7	2.19
		1975	19.9	17.3	-2.6	-13.1	2.10
		1976	18.3	14.5	-3.8	-20.8	2.09
		1977	16.7	16.4	-0.3	-1.8	2.20
		1978	22.9	21.1	-1.8	-7.9	2.08
		1979	20.9	18.8	-2.1	-10.0	2.12
	60	1970	17.5	19.7	2.2	12.6	2.42
		1971	26.5	22.4	-4.1	-15.6	2.23
		1972	22.6	22.4	-0.2	-0.2	2.14
		1973	21.3	18.4	-2.9	-13.6	2.17
		1974	18.4	18.4	0.0	0.0	2.20
		1975	21.5	18.0	-3.5	-16.3	2.09
		1976	22.8	16.5	-6.3	-27.6	2.12
		1977	24.1	18.8	-5.3	-22.0	2.22
		1978	28.6	22.0	-6.6	-23.1	2.34
		1979	29.3	22.9	-6.4	-21.8	2.37
	70	1970	16.4	17.9	1.5	9.1	2.02
		1971	21.6	20.3	-1.3	-6.0	1.99
		1972	21.4	23.3	1.9	8.9	2.06
		1973	22.1	16.6	-5.5	-24.9	2.02
		1974	15.3	16.7	1.4	9.2	2.11
		1975	16.9	17.4	0.5	3.0	2.13
		1976	19.6	16.1	-3.5	-17.9	2.09
		1977	17.2	17.7	0.5	2.9	2.11
		1978	20.8	20.6	-0.2	-1.0	2.14
		1979	17.7	16.6	-1.1	-6.2	2.11
	80	1970	13.0	15.7	2.7	20.8	2.02
		1971	21.4	20.0	-1.4	-6.5	1.99
		1972	18.9	22.0	3.1	16.4	1.97
		1973	16.3	12.5	-3.8	-23.3	2.02
		1974	10.1	14.0	3.9	38.6	2.19
		1975	17.8	17.5	-0.3	-1.7	2.11
		1976	14.2	14.5	0.3	2.1	2.08
		1977	12.7	17.3	4.6	36.2	2.14
		1978	19.0	20.0	1.0	5.3	2.18
		1979	16.5	17.1	0.6	3.6	2.11

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APPENDIX 1
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STATE	CRD	YEAR	YIELD (Q/H) ACTUAL	PRED.	D	RD	S.E. PRED.
N.DAKOTA	90	1970	18.5	15.9	-2.6	-14.1	2.03
		1971	24.8	20.8	-4.0	-16.1	1.98
		1972	21.3	20.8	-0.5	-22.3	2.03
		1973	18.7	11.5	-7.2	-38.5	2.02
		1974	17.1	16.5	-0.6	-33.5	2.23
		1975	17.8	15.5	-2.3	-12.9	2.18
		1976	13.8	11.2	-2.6	-18.8	2.07
		1977	23.1	19.5	-3.6	-15.6	2.13
		1978	22.3	20.0	-2.3	-10.3	2.17
		1979	22.9	19.1	-3.8	-16.6	2.10
STATE MODEL		1970	18.3	19.0	0.7	3.8	1.95
		1971	24.2	21.5	-2.7	-11.2	1.77
		1972	21.5	22.1	0.6	12.8	1.77
		1973	19.9	17.1	-2.8	-14.1	1.69
		1974	15.1	17.2	2.1	-13.9	1.97
		1975	20.4	16.3	-4.1	-20.1	1.65
		1976	20.4	16.8	-3.6	-17.6	1.74
		1977	21.0	16.5	-4.5	-21.4	1.72
		1978	24.7	20.4	-4.3	-17.4	1.84
		1979	24.7	20.2	-4.5	-18.2	1.95
CRDS AGGR.		1970	18.3	18.5	0.2	1.1	
		1971	24.2	21.9	-2.3	-9.5	
		1972	21.5	22.5	1.0	4.7	
		1973	19.9	17.1	-2.8	-14.1	
		1974	15.1	18.0	2.9	-19.2	
		1975	20.4	17.3	-3.1	-15.2	
		1976	20.4	16.5	-3.9	-19.1	
		1977	21.0	18.3	-2.7	-12.9	
		1978	24.7	21.8	-2.9	-11.7	
		1979	24.7	20.8	-3.9	-15.8	

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STATE	CRD	YEAR	YIELD (Q/H) ACTUAL	PRED.	D	RD	S.E. PRED.
MINNESOTA	10	1970	18.4	20.8	2.4	13.0	2.41
		1971	26.9	24.1	-2.8	-10.4	2.28
		1972	24.8	24.4	-0.4	-1.6	2.15
		1973	24.2	22.2	-2.0	-8.3	2.33
		1974	20.9	19.2	-1.7	-8.1	2.20
		1975	21.5	19.6	-1.9	-8.8	2.10
		1976	26.7	19.7	-7.0	-26.2	2.11
		1977	27.4	19.8	-7.6	-27.7	2.24
		1978	28.4	23.7	-4.7	-16.5	2.36
		1979	29.4	24.5	-4.9	-16.7	2.38
	40	1970	22.9	22.2	-0.7	-3.1	2.36
		1971	25.2	23.0	-2.2	-8.7	2.25
		1972	19.3	19.4	0.1	0.5	2.39
		1973	26.3	22.2	-4.1	-15.6	2.15
		1974	21.2	21.9	0.7	3.3	2.16
		1975	18.6	20.4	1.8	9.7	2.10
		1976	13.3	17.7	4.4	33.1	2.13
		1977	27.7	21.0	-6.7	-24.2	2.24
		1978	22.2	22.2	0.0	0.0	2.40
		1979	26.4	24.4	-2.0	-7.6	2.38
STATE MODEL		1970	19.9	19.9	0.0	0.0	3.47
		1971	26.1	22.8	-3.3	-12.6	2.92
		1972	23.1	24.0	0.9	3.9	2.78
		1973	24.7	23.8	-0.9	-3.6	2.70
		1974	21.0	20.4	-0.6	-2.9	2.74
		1975	20.4	21.2	0.8	3.9	2.47
		1976	22.1	19.6	-2.5	-11.3	2.37
		1977	27.4	20.6	-6.8	-24.8	2.31
		1978	26.6	23.7	-2.9	-10.9	2.53
		1979	28.5	24.8	-3.7	-13.0	2.50
CRDS AGGR.		1970	19.9	21.3	1.4	7.0	
		1971	26.3	23.7	-2.6	-9.9	
		1972	23.3	23.0	-0.3	-1.2	
		1973	24.8	22.2	-2.6	-10.5	
		1974	21.0	20.2	-0.8	-3.8	
		1975	20.5	19.9	-0.6	-2.9	
		1976	22.2	19.0	-3.2	-14.4	
		1977	27.5	20.1	-7.4	-26.9	
		1978	26.8	23.3	-3.5	-13.1	
		1979	28.7	24.5	-4.2	-14.6	

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APPENDIX 1
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NORTH DAKOTA AND MINNESOTA
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STATE	CRD	YEAR	YIELD. (Q/H) ACTUAL	PRED.	D	RD	S.E. PRED.
<hr/>							
REGION							
CRDS AGGR.	1970		18.6	19.1	0.5	2.7	
	1971		24.8	22.4	-2.4	-9.7	
	1972		21.9	22.6	0.7	3.2	
	1973		21.1	18.3	-2.8	-13.3	
	1974		16.6	18.6	2.0	12.0	
	1975		20.5	18.0	-2.5	-12.2	
	1976		20.9	17.2	-3.7	-17.7	
	1977		22.9	18.8	-4.1	-17.9	
	1978		25.4	22.3	-3.1	-12.2	
	1979		26.0	21.9	-4.1	-15.8	
STATES AGGR.	1970		18.7	19.2	0.5	2.7	
	1971		24.7	21.9	-2.8	-11.3	
	1972		21.9	22.5	0.6	2.7	
	1973		21.1	18.7	-2.4	-11.4	
	1974		16.7	18.1	1.4	8.4	
	1975		20.4	17.7	-2.7	-13.2	
	1976		20.9	17.6	-3.3	-15.8	
	1977		22.9	17.7	-5.2	-22.7	
	1978		25.3	21.4	-3.9	-15.4	
	1979		25.9	21.7	-4.2	-16.2	

APPENDIX 2 -- Terms and Ranges of Coefficient Values Over Ten Test Years

	MNR MN10, 40, ND30, 60	NDREM ND10, 20, 40, 50, 70, 80, 90	MN STATE	ND STATE
TREND 1	.1325 - .1559	.4320 - .5242	.0403 - .0574	.1313 - .1568
TREND 2	1.0795 - 2.0372	0.6726 - 1.4407	1.2423 - 2.0674	1.0196 - 2.0695
TREND 2SQ	(-0.1434) - (-0.0339)	(-0.0261) - (-0.1173)	(-0.1368) - (-0.0413)	(-0.1545) - (-0.0338)
TX	0.1409 - 0.1949			
TOP	(-0.4602) - (-0.4025)			
TXDS				
CAPRSQ		(-0.00031) - (-0.00009)		0.0038 - 0.0046
DEFSEAS	(-0.0163) - (-0.0132)	(-0.0384) - (-0.0363)		(-0.3066) - (-0.2536)
DEFSEASQ	(-0.00064) - (-0.00058)	(-0.00095) - (-0.00085)	(-0.00026) - (-0.00023)	
PET5SQ		0.00023 - 0.000401		
PET6SQ	(-0.00026) - (-0.00030)		(-0.00053) - (-0.00050)	
PET 7	(-0.6318) - (-0.9421)	(-0.7428) - 0.6636	(-0.14091) - (-0.12792)	(-1.2277) - (-1.0564)
PET7SQ	0.0018 - 0.0029	(-0.0029) - 0.0023		0.00345 - 0.00406
DEF6SQ		(-0.000209) - (-0.000101)		(-0.00033) - (-0.00014)

TERMS

TREND 1, TREND 2 as defined in text

TREND 2SQ = TREND 2**2

TX, TOP as defined in text

PETi = potential evapotranspiration for month i (Thornthwaite, 1945);
i=5(May), 6(June), 7(July)

DEFi = deficit for month i = (PETi - Precip.)

PET1SQ = PET1 **2

DEF1SQ = DEF1 **2

CAPR = cumulative precipitation September through April

CAPRSQ = CAPR **2

DEFSEAS = DEF5 + DEF6 + DEF7 - CAPR

DEFSEASQ = DEFSEAS **2

TXDS = TX *DEFSEAS

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